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Dedicated to mathematics in general and to the following sime in particular: (1) a study of the mon problems of secondary and collegiate mathematics teaching, (2) a true valuation of the discise of mathematics, (3) the publication of high class expository papers on mathematics, (4) the developt of greater public interest in mathematics by the publication of authoritative papers treating its action, humanistic and historical phases.

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In Prospect

Three questions were recently addressed by the Editor of the Mathematics News Letter to each of the forty-eight State Superintendents of Education in the United States and to the Superintendent of Education of the District of Columbia. Up to the present time twenty-nine replies of the forty-nine expected have been received. Long before the date of issue of News Letter No. 1, Volume 9, it is anticipated that all replies will be in the Editor's hands.

The three questions are:

- 1. Has mathematics as a required subject for graduation from high school been eliminated in your state?
- 2. If it has not been eliminated is there a definite prospect that it will be at an early date?
- 3. In your judgment are there good reasons for the hope that mathematics will have increased rather than diminished use in the secondary schools of your state?

Of the replies already received, many have generously exceeded the bounds indicated by the questions. Thus it appears probable that when all are in we shall have a basis for something like a measure of the nation-wide valuations of mathematics presently being made by all of the leading educational administrators in America. To what extent these valuations may be interpreted to mean a lowered or an unchanged rank of the premier science in the estimation of American public opinion it should be possible to determine with a considerable degree of accuracy by a careful gauge of these forty-nine expert official statements.

It is because this journal has always assumed, along with its other functions, the special function of serving the teacher of mathematics in every possible way that we make this editorial announcement of an early forthcoming analysis and interpretation of the data to be returned. The October issue of the News Letter should be anticipated with interest by those of its readers who are seriously concerned with the changed and changing status of mathematics in the secondary school. It is even possible in order that the analysis shall be thorough that it will be necessary to carry it into the November and December issues. What an impressive gamut of opinion will be disclosed to the reader!

Affording a partial pre-view of the forty-nine declarations that are in process of being assembled we quote from a few answers already returned:

Writes M. B. Brown, Deputy State Superintendent of Texas: "In my opinion the first two of the three questions in your letter should be answered in the negative. It is my belief that a return of normal conditions will bring such a demand for technically trained folk that mathematics in the secondary schools will have an increasing popularity."

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From Massachusetts comes the pronouncement of Jerome Burtt, Supervisor of Secondary Education: "Massachusetts has no state regulations with regard to the requirement of any high school subject except American History and Civics. Each community sets up its own graduation requirements. I think many of our high schools no longer require mathematics for graduation and there seems to be an increasing trend in this direction. Many schools which still do require mathematics, give an option between algebra and what is known as general mathematics, thus taking care of many pupils who cannot do the traditional type of algebra successfully."

From far Connecticut comes a word from Commissioner of Education Ernest W. Butterfield: "In Connecticut there has been no general rule which required mathmatics as an element in all high school courses which lead to graduation. It has, however, been a requirement in most of our towns and cities.

"Our high schools have now a very large number of pupils who are not making preparation for college or employment in lines where formal mathematics is a requirement. I anticipate, therefore, that the importance of mathematics of the academic type will decrease in our secondary schools."

G. W. Rosenlof, Director of Secondary Education and Teacher Training of Nebraska writes the following: "In regard to your third question, I have every reason to believe that mathematics has a place in the secondary school curriculum and I would hope an increased rather than a diminished use. This, however, will depend upon the extent to which mathematics instructors are able to relate the subject matter of mathematics definitely to the needs of those being trained to the end that they may more properly and effectively utilize the subject matter in their everyday experiences."—S. T. S.

The Historical Significance of Carl Friedrich Gauss in Mathematics and Some Aspects of His Work

Professor G. Waldo Dunnington, M.A. Kansas City, Missouri

It is difficult to comprehend the significance of a man like Gauss unless we can get the proper perspective as to all the originality and universality of his research, in comparison with his contemporaries and his scientific intercourse with the leading mathematicians and astronomers. Also, we must observe the various fields of science which he pioneered for posterity. Due to limitations of space (and time), all this cannot be done exhaustively in this article; nevertheless, we may indicate his position in the evolution of mathematics.

After the discovery of the differential calculus by Newton and Leibnitz mathematicians were endeavoring to apply this powerful new tool to all the parts of their science. We still admire the creative power of Newton's *Principia* with its mode of presentation reminiscent of the ancients; it was fully elaborated by the three Bernoullis, L'Hospital, and especially Euler. In the middle of the eighteenth century it was France that stood out above the other nations in this respect. One need only mention names such as Clairaut, D'Alembert, Lagrange, Laplace, and Legendre. Germany was still suffering from the consequences of the Thirty Years' War and the unhappy conditions which finally led up to the dissolution of the Holy Roman Empire in 1806. The rise of humanistic studies just before the Reformation period had continued, and although repressed for a while, had survived sub rosa. Literature, history, and the study of antiquities experienced flourishing development.

The study of mathematics and natural sciences lay prostrate. Euler's more than seven hundred memoirs in the transactions of the St. Petersburg Academy were changing all the branches of mathematics. He was temporarily active in Berlin; following him, Lagrange was appointed director of the Berlin Academy by Frederick the Great. Lambert, the physicist and philosopher from Alsace, was active at the same Academy and even today we must grant him a leading rank among the scholars of that time. The important services of Kastner, as a teacher of mathematics, must not be forgotten. In spite of these few exceptions, only the rudiments of mathematics were being taught at the German universities, while physics, in our sense of the term, was

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atter end atter scarcely even thought of. On the other hand, the humanistic studies under men like Christian von Wolff, Friedrich August Wolf, Ernesti, and Heyne were progressing nicely, and Kant was bringing philosophy to the forefront.

Thus we see young Gauss busy with his own pursuits, the study of Euler's and Newton's works. From them he seems to have acquired that characteristic of universality, which binds him closer to them than any other mathematician since his day-in fact he has been called the last of the kind. The entire interest of the earlier mathematicians was concentrated on recognizing natural phenomena in their mathematical regularity. They were unremitting in their efforts to formulate new methods which would lead to this goal. Integral calculus and the calculus of variations may be cited as examples. They were accustomed to attach no independent interest to those methods in themselves, as we do. It was reserved for a later period to make those methods the subject of a purely theoretical research and development. This fact has made mathematics in our time less intelligible to the layman, while at that time the close connection of all theoretical speculation with the great practical questions was much clearer. Universality was fostered by the mode of life then in vogue which made it possible for those scholars to devote themselves with all their might to the urge for research. Indeed it seems that Gauss desired to take Newton and Euler as his models in this respect. At the time of his election to a professorship in Gottingen he said that he placed special importance on being relieved of duties incident to a real teaching position.

Gauss had reason to take this position. His first achievement in astronomy had been the solution of a problem which had previously baffled the most famous. He discovered a method for determining the elliptic orbit of a planet from three observations and calculated the elements of Ceres. He published the method in 1809 in his famous Theoria motus corporum coelestium. How can an orbit thus determined from three observations be harmonized with empirical data—a series of observations? This question had led him in 1795 to the process known as the "method of least squares". Gauss was not the first to publish this method; Legendre discovered it independently and published it in 1806, but it is rightly known as the Gaussian method, because he developed it fully and made it practical.

For twenty years Gauss was occupied intermittently with the triangulation of the Kingdom of Hanover, and because of this survey became the real creator of higher geodesy. It is probable that no one ma nec cal wo by

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has ever worked in this field with the same happy combination of deep mathematical problems and penetrating practical methods. In this connection should be mentioned his invention of the heliotrope and his calculation of Gaussian logarithms. During this period originated his work on cartographic projection—inspired by a prize question sent out by the Danish Academy.

Stimulated by Alexander von Humboldt from 1830 on, he participated in various magnetic experiments. In collaboration with the physicist, Wilhelm E. Weber, who was called to a Gottingen professorship in 1831, he succeeded in developing the theory of terrestial magnetism into a new science. Gauss now showed how to present these results by means of a mathematical formula, the declination and inclination of the magnetic needle, the intensity at every point on the earth. He introduced the so-called absolute units (i. e., independent of time and place). The importance of determining these constants led to his discovery of the bifilar magnetometer and several other instruments.

His "Theory of Terrestial Magnetism" appeared in 1839. Such men as Tobias Mayer and Hansteen had tried to explain this phenomenon on the basis of single magnets in the interior of the earth. Gauss conceived of a power which originates in a similar manner in the activity of magnetic molecules in the earth. Such forces are determined by their so-called potential, and he showed how this potential can be numerically calculated. Along with Laplace he became the founder of the theory of the potential. This work was of importance in later mathematical physics, as developed by Faraday, Maxwell, and Hertz, whose results led up to modern radio.

In this period falls his important research on the equilibrium of fluids, electrodynamics, and capillarity. Before the Royal Society of Gottingen he presented in 1843 his "Dioptric Studies on Systems of Lenses" and won a new harvest in a field which the unthinking believed had been already exhausted by Cotes, Euler, Lagrange, and Mobius. A most practical discovery had already occurred at Easter time, 1833. Gauss and Weber produced the first electromagnetic telegraph by stretching a wire 8,000 meters long, from the observatory to the physics laboratory. They had devised an alphabet, or code, and soon the later telegraphic advances in Europe and America were closely connected with this early development, especially the transatlantic line. As a physicist, Gauss may well be compared with Newton and Galileo.

Although so open and animated about these practical results, he was extremely reticent about his purely theoretical research, rightfully

put a higher value on it, and had difficulty in initiating his more apt students into it. His memoirs were models of presentation, the rigor of his proofs became proverbial. This "artistic structure" often makes it difficult to penetrate his writings. As a youth his mind was so flooded with ideas that there was not sufficient time to write them out as a "completed work of art," in accordance with his desires. This accounts for the fact that much perished at his death, and many of his discoveries (found among his papers) saw the light of day only after that date.

His doctoral thesis (1799) constituted the first rigorous proof of the fundamental theorem of algebra, viz., that every equation of degree n in one variable has n roots, or, better, that every integral rational algebraic function can be factored into real factors of the first and second degree. In later life he gave several alternative and improved proofs, five in all. In 1799 he made use of complex and imaginary numbers, representing them in a plane.

The outstanding work of his life, the Disquisitiones arithmeticae (1801) founded a new epoch in the theory of numbers. Legendre's Theorie des nombres, appearing about the same time, left off where Gauss began. His discovery in 1796 of the method of inscribing a regular polygon of seventeen sides in a circle with compasses and straightedge, his general theory of circle-division and primes are so famous as scarcely to need repetition here. The theory of determinants and algebraic forms goes back to him. Since his day the theory of numbers has been especially cultivated in Germany, beginning with such men as Eisenstein, Jacobi, and Dirichlet.

Jacobi's name always comes to mind when one mentions a part of mathematics characteristic of the nineteenth century—the theory of elliptic and Abelian functions in connection with the theory of functions created by Riemann. From papers left at his death it was revealed that Gauss a generation earlier (1799) had been fully conscious of the idea and its importance, from which Abel's and Jacobi's achievements originated. He never found the time to complete his theory on paper.

The geodetic survey led him about 1827 to a consideration of the general properties of curved surfaces, to the question of the representation of one surface on another, so that the smallest parts shall be similar. Here he treats in particular the case of a surface which can be brought to the shape of another given surface merely by bending, and, introducing the measure-of-curvature concept, shows that under these circumstances it must have the same value for both surfaces.

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Gauss was undoubtedly in possession of the fundamental facts underlying the later development of non-euclidean geometry. Unfortunately he never published anything on the subject, and put down very little on paper. He did encourage the son of his old friend Bolyai, also his student Riemann, and gave recognition to Lobachevsky in his research and publication on the subject. There has been much controversy on the exact part played by Gauss in the discovery of non-euclidean geometry, but this is not the place to go into it.* It is curious, however, to note that Lobachevsky, Bolyai, and Riemann all had personal connections of one kind or another with Gauss.

The first portion of this article was intended to show how Gauss ties up with the great mathematicians of the seventeenth and eighteenth centuries in the effort to subject all parts of the disciplines understood by a wider circle to his forceful analytical power. The latter part shows how in his purely theoretical research he belongs entirely to modern times, indeed to what extent his ideas are still regnant or the subject of great interest in science and mathematics. To his contemporaries, perhaps, he seemed rather isolated, especially in old age. Today we realize that in the quietude he was pioneering for them, that the driving elements of mathematical research have followed in his footsteps, but that he has never had any real successor. It is a fact that we better appreciate any great personality the farther we progress, and connect their endeavors with their own evolution. Of course, Gauss enjoyed during his own lifetime the happiness due to a consciousness of full recognition by his contemporaries. To us he appears even greater if we attempt to be just to him in all his historical significance. He was a master in the deepest and most abstract questions of knowledge, with genuine originality and versatility in the application of it.

* The present writer plans to devote an entire chapter to this question in a full biography of Gauss, as yet uncompleted.

On the Mathematics of Stock Quotations

By C. D. SMITH

The New Deal in American finance dates from approximately March 1, 1933. Probable upward trends in values and the probable laws of fluctuation by which they may be expected to rise presents a

question of current interest. We have progressed far enough now to make preliminary studies from available data, for example the quotations from records of the New York Stock Exchange. Such quotations indicate consolidated opinion as to what the progress of values should be before the history of a period actually records the values. One naturally wants to know what degree of accuracy may be expected of such predictions and to what extent one may be guided by purely mathematical considerations in dealing in stocks. The object of this paper is to illustrate a method of mathematical analysis applicable to such problems. A random sample of daily maximum fluctuations is taken from records of the New York Stock Exchange from March 1 to July 1, 1933 as reported by the Commercial and Financial Chronicle. For definiteness we will develop the problem under the following main divisions: The Sample; The Distribution, Mathematical Constants; A Theoretical Population; Conclusions.

The Sample. In selection of data it is necessary to so limit the population and so choose each datum that we may assume with a high degree of probability that the data represent a random sample of the population from which the sample is drawn. Observing that quotations are grouped as Railroads and Industrials and assuming that transportation is more indicative of general conditions than other values and are more nearly comparable than mixed quotations we select one thousand daily fluctuations in railway stocks as a sample. Equal numbers of items are taken each day ranging at random from the highest to the lowest priced stocks and from the most active to those for which there are no daily bids. In this way we seek to justify the assumption that the sample is typical of daily fluctuations for the period March 1 to July 1, 1933. Here the following questions are suggested.

- 1. What is a fair estimate of the expected value of a maximum daily fluctuation?
- 2. What is the expected variance of daily fluctuations?
- 3. What general form may be expected of the range of fluctuations?
- 4. What general principle should guide buying and selling in a rising market?
- 5. What is the probability that a fluctuation greater than a given amount may occur?

These are some of the questions considered in the following.

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The Distribution. It is necessary to arrange the data of the sample in classes for convenience in making computations. Since the range is from zero to 6.25 points recorded in eighths of a point we select as class interval two eighths. Representing the class by t_i and corresponding frequency by f_i the distribution of fluctuations in eighths of a point is as follows:

A slight error is introduced here due to grouping the data which will be reflected in the values of moments. We will not apply the usual method of correction known as Shepard's corrections due to the high frequency at one end of the distribution. For convenience in computing the necessary statistical values we select 10 as origin and indicate the classes by \mathbf{u}_i for which the class interval 2 is unit. The distance from the selected class to the arithmetic mean is designated by \mathbf{u}_i , the moments about origin 10 by \mathbf{v}_i , and the corresponding moments about the mean by \mathbf{u}_i . The variance of the sample is represented by \mathbf{u}_i and the standard deviation or square root of the variance by \mathbf{u}_i . We also represent skewness by S and kurtosis by K. Texts in mathematical statistics may be consulted for origin of the formulas used in the next section for computing the necessary values.

Mathematical Constants.

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$$u = (1/N)\Sigma u^3f = -.054$$
. $v_2 = (1/N)\Sigma u^2f = 18.765$.

$$v_3 = (1/N) \Sigma u^3 f = 109.233. v_4 = (1/N) \Sigma u^4 f = 1842.763.$$

$$\frac{\mu}{2} = v_2 - \overline{u}^2 = 18.763$$
 (the variance of the sample).

$$\frac{\mu}{3} = v_3 - 3v_2\overline{u} + 2\overline{u^2} = 112.273.$$

$${}^{\mu}_{4} = v_{4} - 4v_{3}\overline{u} + 6v_{2}\overline{u}^{2}3\overline{u}^{4} = 1866.685.$$

$$\theta = \frac{\mu^{1/2}}{2} = 4.33$$
 (the standard deviation of the sample).

$$\alpha_3=\frac{\mu}{3}/6^3=1.4.$$
 $\alpha_4=\frac{\mu}{4}/6^4=5.3.$ $S=\alpha_3/2=.7$ (skewness of the sample).

$$K = (\alpha_3 - 3)/2 = 1.15$$
 (kurtosis of the sample).

It is especially interesting at this point to consider the implications which attend the above values. In the first place if you plot the points indicated by class and corresponding frequency you will observe that a very large portion of the area under the polygon is below 10 with mode or highest point at two. With the range given in eighths of a point we see at a glance how small are a great majority of daily fluctuations. The arithmetic average computed from the value of $\bar{\mathbf{u}}$ is 9.892, or a little more than one point. The variance $\frac{\mu}{2}$ being approximately one-third of the total range indicates a relatively high degree of concentration about the mean. The value .7 for skewness reflects the location of high frequency at the lower bound and the relatively long low spread of the sample to the right of the mean. The positive kurtosis is a significant value and is caused by the concentration of data into relatively few classes to left of the mean so as to produce a decided peak in the polygon. With these special characteristics the whole distribution is fairly smooth and regular. chief value of these characteristics lies in the clue which they furnish to the problem of the next section in which we seek a general distribution of the population. We are now ready to attack the general problem of establishing a general formula or curve which represents the population from which the sample is drawn and to make estimates of the probable size of fluctuations.

A Theoretical Population. There is no mathematical criterion for selecting a formula or curve which best fits a given sample. characteristic values which we have computed serve as a general guide. With a formula once selected it is general practice to determine the proper values of parameters by the so called method of least squares. By this we mean that the sum of squares of the differences between observed and corresponding theoretical frequency shall be a minimum. A more satisfactory result is sometimes attained by use of either the Pearson System or the Gram-Charlier System of frequency curves. Reference to the mathematical development of these systems may be found in the Notes of the Carus Mathematical Monograph, No. 3. Referring again to the values computed in the previous section we note the skewness with high frequency near zero as natural lower bound of the sample and the mode notably near zero. These characteristics suggest the Gram-Charlier Series, Type B, as a likely representative of daily market fluctuations. With this agreed we proceed to set up the

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$$F(x) = C_0 \psi(x) + C_1 \Delta \psi(x) + \dots + C_n \Delta^n \psi(x) + \dots,$$

where $\psi(x) = e^{-\lambda} \lambda^4/x!$. Here λ is a parameter which we shall take equal to the arithmetic mean of the sample and $\triangle^n \psi(x)$ is the nth finite difference of $\psi(x)$. The series is known to converge very rapidly so as to justify the use of the first three terms as approximation. Hence we use

$$F(x) = C_0 \psi(x) + C_1 \Delta \psi(x) + C_2 \Delta^2 \psi(x).$$

The coefficients in terms of the observed distribution are found to be $C_0=1$; $C_1=0$; $C_2=(\frac{\mu}{2}-\lambda)/2$. Substituting these values we have for a law of the population,

$$F(x) = \psi(x) + 1/2(\frac{\mu}{2} - \lambda) \triangle^2 \psi(x).$$

By using the indicated values of $\psi(x)$, $\frac{\mu}{2}$, and λ the curve may be plotted.

The next question of interest is that of estimating the probability of a daily fluctuation of given value. For this purpose we will use the method of Generalized Tchebycheff Inequalities. A detailed account of such formulas appears in my paper in The American Journal of Mathematics, Vol. LII, No. 1, January, 1930. If we ask for the probability that a daily fluctuation may deviate from a selected value by more than a given amount d we may of course compute the corresponding frequency from the theoretical formula when possible and divide the result by the total frequency. The result is an estimate of the true probability. In most cases it is easier to estimate the probability by means of boundary values and we shall now illustrate the method.

We first use the formula $P_d \gg 1/t^n$ where $d = t \frac{\mu^{1/n}}{n}$ and for example take as unit of measure $\frac{\mu^{1/n}}{4} = 6.5$. If t = 3 we are asking for the probability P_d of a deviation from the average of approximately five points. The formula gives $P_d \gg 1/81$. As a closer estimate of the probability we have the formula $P_d \gg \beta_{2\tau-2}/(t/\theta)^{2\tau}$ where $d = t \ell$, $t \theta = 2r[t^{2\tau+1} - (C\theta)^{2\tau+1}]/(2r+1)[t^{2\tau} - (C\theta)^{2\tau}]$ and $c \ell = 0$ distance from origin to the mode of the distribution. Since $\ell = 4.33$ and the distance to mode is 3.89 we have approximately c = .9. Assuming the same deviation of five points approximately we find t = 4.5 and using 4th moments again we have r = 2, $\beta_2 = \frac{\mu}{4}/6^4 = 5.3$, $\theta = .9$. The result of substitution is $P_d \gg .008$.

Taking this latter estimate we say that a daily fluctuation of as much as five points from the average would be expected to occur not more than 8 times in 1,000. A somewhat closer estimate may be found by moving the origin a distance $h = \frac{\mu}{2}/d$ to the left of the mean and using moments about the new origin. However since we now know the probability is less than .008 we would hardly care to pursue the case further. As a check on the general problem one might try one of Pearson's Type curves and check the result with the above form of the Type B series.

Conclusions. For convenience we list certain observations which are suggested by the results of the paper.

- One should remember that the sample used represents a rather rapidly rising market and the range of fluctuations is correspondingly wider than would ordinarily occur. Such renders the results more conservative for a normally fluctuating market.
- The expected fluctuation of a stock taken at random from the market is slightly more than a point while the most probable value is one fourth of a point.
- 3. The Gram-Charlier Series, Type B, is the general form of the population of absolute values of maximum daily fluctuations in stock prices.
- 4. Generalized Tchebycheff formulas give fair approximations to the value of the probabilities for fluctuations of given value.
- 5. One is not justified in buying or selling stocks solely on a statistical basis. So many hidden influences affect sudden market fluctuations in individual stocks as to render estimates on the basis of the average inadequate for prediction of rise or fall. It does seem fair to conclude that one who has a particular stock to sell would be justified in acting on a fluctuation above the average. Or if one is ready to buy and has settled on the basic values of some stock a negative fluctuation more than the average would present a good probability of safety. Such conditions as consecutive runs in rise or fall are not included in the study.

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Edited by P. K. Smith

Felix Klein, Elementary Mathematics from an Advanced Standpoint. Translated from the third German edition (1924) by E. R. Hedrick and C. A. Noble. New York, Macmillan, 1932. 274 pages.

The writers of the textbooks in elementary mathematics, from arithmetic through calculus, have slurred over or omitted many essential steps in the logical development of their theory. It is possible for them to do this because our schooling methods encourage students to be receptive and uncritical. It is thought advisable for them to do this because the suppressed developments are too long and difficult, or too impractical, or too abstract to be appreciated by immature minds. It seems generally to be believed that, since a student's first grasp of a new idea is necessarily incomplete and inaccurate, an effective presentation of that idea must likewise be incomplete and inaccurate. The number system, especially irrational numbers, the postulational foundations of a theory, the simple properties of primes, exponents, logarithms, continuity and differentiability of functions, complex numbers, the existence of the numbers e and II, not to mention their transcendental character, impossible ruler and compass constructions, solutions of algebraic equations in terms of radicals, are but examples of topics which crop up in elementary mathematics and are treated like orphans.

Certainly there is practical justification for some measure of compromise of the strictly formal and logical method in teaching elementary mathematics. We must strike somewhere between the absurd extreme of beginning arithmetic with the postulational foundation of the number system and the other extreme of sinking into complete intellectual dishonesty by omitting all deductive proofs. Klein, in his Elementary Mathematics From an Advanced Standpoint, seems to think that we still lean too far toward the latter extreme. The book is written principally for young teachers of mathematics, but it is a book on the subject matter, not the pedagogy, of mathematics. The topics discussed include the list mentioned above. A topic like logarithms, for instance, is discussed as it is taught (as of 1908 or 1924 in Germany), as it developed historically, as it might be taught to elementary students, and as it is developed in modern function theory.

The book has a unique composition. Many of its statements are proved in completely rigorous detail. Other proofs are merely started by the author who indicates the direction of further investigation and refers the reader to a reliable source. Formal proofs, however incontrovertible and elegant, do not satisfy the unsophisticated student when he asks why. Klein's rich historical introductions to various topics, his frequent appeal to intuition and to geometric perception help to provide the motivation which even the young teacher may need. And they make the book about as readable, in the popular sense, as accurate mathematics can be. It is thoroughly stimulating and could be read with pleasure and profit by any teacher of mathematics. The translators have done a notable service.

The principal drawback to the translation is that for completeness the book leans heavily upon other words, particularly Weber and Wellstein's Encyklopadie der Elementarmathematik and Tropfke's Geschichte der Elementarmathematik and these, alas, are still in German. Furthermore, the book was written years ago with the German school system in mind. Could not some American Mathematician of recognized ability do for American mathematics teaching a greater service by writing for the American teacher as Professor wrote for the German teacher?

W. L. Duren, Jr.

Tulane University.

Problem Department

Edited by T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

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LATE SOLUTIONS

A solution for number 35 has been received from an eleventh grade pupil of Francis P. Nash, Groton, Massachusetts.

SOLUTIONS

No. 51. Proposed by Henry Schroeder, Louisiana Polytechnic Institute.

Determine a point whose distances from three fixed points should have a minimum sum.

Remark by A. F. Moursund, University of Oregon.

For a solution of this problem, see "Goursat-Hedrick, Mathematical Analysis, Vol. 1, Paragraph 2."

Bibliographical Note by Nathan Altshiller-Court, University of Oklahoma.

This problem has a long and honorable history. It was proposed by Pierre Fermat (1601-1665; Oeuvres, p. 153, Paris, 1891) to E. Torricelli (1608-1647) who solved it in three different ways. Torricelli in turn proposed it to Vincenzo Viviani (1622-1702) who also found a solution (De Maximis et minimis, 1658). Fermat's problem and its various generalizations have since called forth the efforts of many mathematicians, among them some with very illustrious names. Of the generalizations of Fermat's problem one of the most recent ones is due to Gino Loria (Mathesis, 1899, pp. 131-136).

J. Bertrand (1822-1900) has discussed Fermat's problem as a case of a minimum of a function of two variables (Journal de Mathematiques pures et appliquees, 1843, p. 155). This discussion has since passed into many books on the differential calculus, as E. Goursat's Cours d'Analyse, Vol. I; English edition by Hedrick, 1904, p. 130.

Several simple solutions of Fermat's problem may be found in the American Mathematical Monthly, Vol. XXVII, (1920), p. 38, Q. 2742. See also Educational Times, Reprints, Vol. LX (1894), p. 83, Q. 11895. J. J. Milne, Companion to weekly problem papers, p. 36. MacMillan, 1888; J. W. Russell, A sequel to elementary geometry, p. 134, Oxford, 1913.

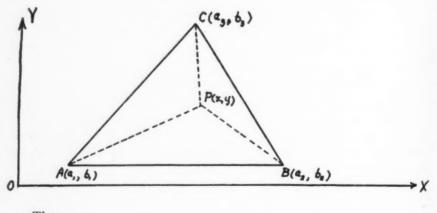
The point which solves Fermat's problem has the property that the sides of the given triangle subtend at this point equal angles, and on this account this point is often referred to as the "isogonic center" of the triangle (Roger A. Johnson, Modern Geometry, p. 221, Houghton Mifflin Co., 1929).

The isogonic center comes also to light in connection with the three equilateral triangles constructed on the sides of the given triangle as bases (Nathan Altshiller-Court, College Geometry, pp. 105-107; Johnson Publishing Company, Richmond, Va., 1925).

A Solution by H. F. S. Jonah, Purdue University, West Lafayette, Indiana.

Geometrically, we can show that the point P must lie in the plane determined by the three given points, if, we wish a minimum sum. We will further assume that the three given points do not lie on a line. In the case that the three points lie on a line one can easily determine the location of P.

We will, then, consider the problem: Being given a triangle ABC, to find a point P, in the plane of the triangle, such that the sum of the distances to the three vertices of the triangle is a minimum, i e, PA+PB+PC=min.



Then

1)
$$z = \sqrt{(x-a_1)^2 + (y-b_1)^2 + \sqrt{(x-a_2)^2 + (y-b_2)^2}} + \sqrt{(x-a_3)^2 + (y-b_3)^2}$$

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This function is continuous and by certain theorems of Real Variables we can show that this function actually takes on its minimum. The equation (1) represents a surface S, which is entirely above the xy-plane. Hence, our problem reduces to the question of finding the point on this surface nearest to the xy-plane.

Taking the partial derivatives of z of first order, first with respect to x, then with respect to y, we obtain

2)
$$\frac{\partial z}{\partial x} = \frac{x - a_1}{\sqrt{(x - a_1)^2 + (y - b_1)^2}} + \frac{x - a_2}{\sqrt{(x - a_2)^2 + (y - b_2)^2}} + \frac{x - a_3}{\sqrt{(x - a_3)^2 + (y - b_3)^2}}$$
3)
$$\frac{\partial z}{\partial y} = \frac{y - b_1}{\sqrt{(x - a_1)^2 + (y - b_1)^2}} + \frac{y - b_2}{\sqrt{(x - a_2)^2 + (y - b_2)^2}} + \frac{y - b_3}{\sqrt{(x - a_2)^2 + (y - b_2)^2}}$$

Now, by well known methods of Real Variables, we can show that these derivatives are continuous except in the neighborhoods of the points A, B, C, where they are discontinuous.

In order to solve the equations $\partial z/\partial x = 0$, $\partial z/\partial y = 0$ for the point P, let us write them in the form

4)
$$\frac{x-a_1}{\sqrt{(x-a_1)^2+(y-b_1)^2}} + \frac{x-a_2}{\sqrt{(x-a_2)^2+(y-b_2)^2}}$$

$$= -\frac{x-a_3}{\sqrt{(x-a_3)^2+(y-b_3)^2}}$$

$$\frac{y-b_1}{\sqrt{(x-a_1)^2+(y-b_1)^2}} + \frac{y-b_2}{\sqrt{(x-a_2)^2+(y-b_3)^2}}$$
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$$= - \frac{y - b_3}{\sqrt{(x - a_3)^2 + (y - b_3)^2}}.$$

Squaring and adding and making use of the analytical properties of the terms, we obtain

$$6) \quad 1+2 \left| \begin{array}{c} (x-a_1)(x-a_2) \\ \hline \sqrt{x(-a_1)^2 + (y-b_1)^2} \sqrt{(x-a_2)^2 + (y-b_2)^2} \\ \\ + \frac{(y-b_1)(y-b_2)}{\sqrt{(x-a_1)^2 + (y-b_1)^2} \sqrt{(x-a_2)^2 + (y-b_2)^2}} \right| = 0 \\ \end{array}$$

Let us denote by α and α^{I} the angles which PA makes with the x- and y-axis, respectively. β and β^{I} the angles which PB makes with the x- and y-axis, respectively. Then (6) becomes

6')
$$1+2[\cos\alpha\cos\alpha^1+\cos\beta\cos\beta^1]=0$$

Now if ω is the angle between AP and PB, namely \angle APB.

Then

$$\cos \omega = \cos \alpha \cos \alpha^1 + \cos \beta \cos \beta^1$$

And (6') becomes

$$1 + 2 \cos \omega = 0.$$

Therefore

7)
$$\cos \omega = -\frac{1}{2}.$$

Thus

$$\omega = 120^{\circ}$$
.

In the same way we can show that the angles APC and BPC are each equal to 120° .

By geometrical considerations we can show, if the sum z is a minimum, that the point P must lie inside the triangle. In the case where we have a triangle with an angle = or $>120^\circ$, there will be no point P inside the triangle, which will give us a minimum as required by 7) but that the point P in this case will fall at the vertex of the triangle which has the angle = or $>120^\circ$.

Testing the case, where P is inside the triangle (i. e. where all the angles of the triangle are $<120^{\circ}$), geometrically we can show that the sum PA+PB+PC is less than the sum of any two sides of the triangle and therefore a minimum. By similar considerations we can show, geometrically, in the other case that the sum is still a minimum when the point P falls on the vertex of a triangle with an angle = or $>120^{\circ}$.

Also solved by A. W. Randall, Prairie View, Texas, and R. B. Thompson, Beaver Crossing, Nebraska.

No. 53. Proposed by T. A. Bickerstaff.

Sum the series

 $\sin \alpha + 1/2 \sin 2\alpha + 1/2^2 \sin 3\alpha + 1/2^3 \sin 4\alpha + \dots$ to infinity.

Solution. By A. F. Moursund, University of Oregon.

Let
$$C(r,\alpha) = 1 + r \cos \alpha + r^2 \cos 2\alpha + \dots$$
,

$$S(r,\alpha) = r \sin \alpha + r^2 \sin 2\alpha + \dots$$

then
$$C(r,\alpha) + iS(r,\alpha) = 1 + re^{i\alpha} + r^2e^{2i\alpha} + \dots$$

When $|r| \angle 1$, by the well known formula for the sum of an infinite geometric series,

$$C(r,\alpha) + iS(r,\alpha) = \frac{1}{1 - re^{i\alpha}} = \frac{1}{1 - r\cos\alpha - ir\sin\alpha}$$
$$= \frac{1 - r\cos\alpha + ir\sin\alpha}{1 - 2r\cos\alpha + r^2\cos^2\alpha + r^2\sin^2\alpha}.$$

Hence

$$C(r,\alpha) = \frac{1 - r \cos \alpha}{1 - 2r \cos \alpha + r^2}, S(r,\alpha) = \frac{r \sin \alpha}{1 - 2r \cos \alpha + r^2}$$

It follows that

$$\sin \alpha + 1/2 \sin 2\alpha + 1/2^2 \sin 3 \alpha + \dots$$

$$= 2 S(1/2,\alpha) = \frac{\sin \alpha}{1 - \cos \alpha + 1/4} = \frac{4 \sin \alpha}{5 - 4 \cos \alpha}$$

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Also solved by A. W. Randall, H. F. S. Jonah, Richard A. Miller, and the proposer.

No. 60. Proposed by James A. Marlowe, University of Mississippi.

A pole with upper and lower radii of a and b respectively, is entirely covered by wrapping with a strip of ribbon two inches wide, the upper edge of the ribbon starting at the bottom of the pole and stopping at the top of the pole. A bird is attached to this upper edge, and it begins to fly so as to unwind the ribbon and always keep the upper edge of that part unwound always drawn horizontally. If the length of the pole is h, find the distance traveled by the bird in unwinding the strip of ribbon.

No. 61. Proposed by A. F. Moursund, University of Oregon, Eugene, Oregon.

Show that

$$\sum_{i=2}^{P+1} \frac{(-1)^{i}(P+1)!}{i(i-2)!(P+1-i)!} = 1.$$

No. 62. Proposed by A. F. Moursund, University of Oregon.

Given

(1) q(t) is non-negative and monotone increasing on (0,1):

(2)
$$\int_{0}^{1} q(t)dt = 1$$

Where

$$Q(h) \ = \ \int_{1-h}^{1} \ q(t) \ dt, \ h = \Pi/ns \ \gg 1$$

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